DISTRIBUTION OF LIQUID OVER A RANDOM PACKING. VIII.*
DISTRIBUTION OF THE DENSITY OF WETTING IN A PACKING FOR AN ARBITRARY TYPE OF INITIAL CONDITION

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Received December 21st, 1972

A solution is given to the problem of the distribution of the density of wetting in a cylindrical randomly packed column for the boundary condition given by Eq. (*2*). The generality of the solution is limited by the initial condition which must be axially symmetric and thus preserves the twodimensional character of the problem. The solution is then given explicitly for several selected types of initial wetting and the computed curves are compared with the experimentally determined distribution in a 291 mm column packed with four different packings. The distributors used in experiments are: the central "point" source and the disc distributor. A very good agreement is found in both cases.

In the preceding paper of this series[1] solution has been given to the so-called diffusion equation, which may be written in dimensionless cylindrical form as

$$\frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r} = \frac{\partial f}{\partial z}, \qquad (1)$$

for two important types of initial wetting of the packing: the uniform and the wall wetting. The necessary boundary condition has been proposed earlier in the dimensionless form as

$$-\frac{\partial f}{\partial r} = B(f - CW), \quad r = 1 \qquad (2)$$

and verified[2] on an extensive set of experimental data in a wide range of liquid flow rates, for several liquids and packings. One can thus expect that solution to Eq. (*1*) with the boundary condition (*2*) should be a useful tool in description of the processes on random packings, as has been shown already *e.g.* in a paper[3] dealing with the effect of liquid distribution on liquid hold-up in packed columns. The aim of this

* Part VII: This Journal *38*, 1012 (1973).

paper is to obtain a solution to Eq. (*1*) for an axially symmetric but otherwise general condition, *i.e.* for an arbitrary type of a distributor preserving the axial symmetry, and to verify selected types experimentally.

THEORETICAL

One can easily find that the solution to the cylindrical diffusion Eq. (*1*) satisfying the axial symmetry condition has the form

$$f = \sum_{n} A_n \, J_0(q_n r) \exp\left(-q_n^2 z\right). \tag{3}$$

If the boundary condition, Eq. (*2*), is to be satisfied the eigenvalue q_n must satisfy the following equation

$$[(2C/q_n) - (q_n/B)] \, J_1(q_n) + J_0(q_n) = 0\,. \tag{4}$$

It remains now to expand the initial condition as a series of the Bessel functions and from comparison with the solution (*3*) at $z = 0$ to express the so far unknown coefficients A_n. Let us suppose that the initial condition determining the distribution of liquid on the top of the packing is given by a function $\varphi(r)$. Of this function we shall only suppose that it is symmetric (even) with respect to r and has a zero in the point $r = 1$: $\varphi(1) = 0$. Thus we search for the coefficients of

$$\varphi(r) = \sum_{n} A_n \, J_0(q_n r)\,, \tag{5}$$

where q_n are the roots of Eq. (*4*).

To determine the coefficients of the expansion (*5*) one needs to evaluate the integral $\int_0^1 r \, J_0(q_n r) \, J_0(q_m r) \, dr$, where both q_n and q_m are the roots of Eq. (*4*). This is achieved with the aid of the relation following from the properties of the Bessel equation and which can be found in the literature, *e.g.*[4]

$$(q_n^2 - q_m^2) \int_0^1 r \, J_0(q_n r) \, J_0(q_m r) \, dr = q_m \, J_0'(q_m) \, J_0(q_n) - q_n \, J_0(q_m) \, J_0'(q_n)\,. \tag{6}$$

Making use of $J_0'(x) = - J_1(x)$ and substituting from Eq. (*4*) we get

$$\int_0^1 r \, J_0(q_n r) \, J_0(q_m r) \, dr + J_0(q_n) \, J_0(q_m) \, 2C/[(2C - q_n^2/B)(2C - q_m^2/B)] = 0\,. \tag{7}$$

For individual A_n we then have

$$\int_0^1 r\varphi(r)\,J_0(q_n r)\,dr = A_n\left[\int_0^1 r\,J_0^2(q_n r)\,dr + 2C\,J_0^2(q_n)/(2C - q_n^2/B)^2\right]. \qquad (8)$$

The integral on the right hand side may be found also in standard textbooks[4]

$$\int_0^1 r\,J_0^2(q_n r)\,dr = [(J_1^2(q_n) + J_0^2(q_n)]/2\,. \qquad (9)$$

Changing to a single Bessel function with the aid of Eq. (4) and substituting into Eq. (8) we obtain for A_n finally

$$A_n = \{2(q_n^2/B - 2C)^2/[((q_n^2/B - 2C)^2 + q_n^2 + 4C)\,J_0^2(q_n)]\}\int_0^1 r\varphi(r)\,J_0(q_n r)\,dr\,. \qquad (10)$$

The coefficient A_0 can be found easily from the balance on liquid and the limiting form of the boundary condition (2) at $z = \infty$

$$A_0 = C/(1 + C)\,. \qquad (11)$$

Eqs (3), (10) and (11) were used to solve the distribution of the density of wetting in a cylindrical column wetted by a disc distributor of the radius r_1. The mathematical formulation of such initial condition is then: $f = 1/r_1^2$ for $0 \leqq r \leqq r_1$; $f = 0$ for $r_1 < r \leqq 1$. An explicit form of the solution is given in Table I together with certain additional properties of the distribution such as the gradient of the density of wetting in the proximity of the wall, *i.e.*: $(\partial f/\partial r)_{r=1}$, and the magnitude of the wall flow W. From the solution for the disc distributor one can easily change to that for a central point source by the limit $r_1 \to 0$. The mean density of wetting, f_0, appearing in the dimensional form of the solution is replaced in this case by $Q/(\pi R^2)$. This solution is also given in the Table. The third solution tabulated in Table I is that for an annular distributor of the radii r_1, r_2. The initial condition is then: $f = 1/(r_2^2 - r_1^2)$ for $r_1 \leqq r \leqq r_2$; $f = 0$ for $0 \leqq r < r_1$ and $r_2 < r \leqq 1$. The solution can be obtained again from Eqs (3), (10) and (11), or as a difference of solutions for two discs of the radii r_1 and r_2 which is admissible in view of the linearity of the problem. Before substraction, however, both solutions must be multiplied by appropriate weights given by the area of the distributor in order to preserve the overall balance of liquid.

From the solution for the annulus one can easily change in the limit $r_2 \to r_1$ to the solution for a circular distributor of the radius r_1. The mean density of wetting,

TABLE I

Solutions under Selected Initial Conditions

Central point source	$f = \frac{C}{1 + C} + \sum_n \frac{[(q_n^2/B) - 2C]^2 \, J_0(q_n r) \exp(-q_n^2 z)}{\{[(q_n^2/B) - 2C]^2 + q_n^2 + 4C\} \, J_0^2(q_n)}$
$f = \begin{cases} \nearrow \infty & r = 0 \\ \searrow 0 & 0 < r \leqq 1 \end{cases}$	$(\partial f/\partial r)_{r=1} = - \sum_n \frac{q_n^2[(q_n^2/B) - 2C] \exp(-q_n^2 z)}{\{[(q_n^2/B - 2C]^2 + q_n^2 + 4C\} \, J_0(q_n)}$
$2\int_0^1 rf \, dr = 1, \; z = 0$	$W = \frac{1}{1 + C} - \sum_n \frac{2[(q_n^2/B) - 2C] \exp(-q_n^2 z)}{\{[(q_n^2/B) - 2C]^2 + q_n^2 + 4C\} \, J_0(q_n)}$
Disc distributor	$f = \frac{C}{1 + C} + \frac{1}{r_1} \sum_n \frac{2[(q_n^2/B) - 2C]^2 \, J_0(q_n r) \, J_1(q_n r_1) \exp(-q_n^2 z)}{\{[(q_n^2/B) - 2C]^2 + q_n^2 + 4C\} \, q_n \, J_0^2(q_n)}$
$f = (1/r_1)^2, \; 0 \leqq r \leqq r_1$	$(\partial f/\partial r)_{r=1} = - \frac{1}{r_1} \sum_n \frac{2q_n[(q_n^2/B) - 2C] \, J_1(q_n r_1) \exp(-q_n^2 z)}{\{[(q_n^2/B) - 2C]^2 + q_n^2 + 4C\} \, J_0(q_n)}$
$f = 0, \; r_1 < r \leqq 1$	$W = \frac{1}{1 + C} - \frac{1}{r_1} \sum_n \frac{4[(q_n^2/B) - 2C] \, J_1(q_n r_1) \exp(-q_n^2 z)}{\{[(q_n^2/B) - 2C]^2 + q_n^2 + 4C\} \, q_n \, J_0(q_n)}$

Annular distributor

$$f = \frac{1}{r_2^2 - r_1^2}, \quad r_1 \leqq r \leqq r_2$$

$$f = 0 \begin{cases} 0 \leqq r < r_1 \\ r_2 < r \leqq 1 \end{cases}$$

$$f = \frac{C}{1 + C} + \frac{1}{r_2^2 - r_1^2} \sum_n \frac{2\,[(q_n^2/B) - 2C]^2\, J_0(q_n r)\, [r_2\, J_1(q_n r_2) - r_1\, J_1(q_n r_1)] \exp(-q_n^2 z)}{\{[(q_n^2/B) - 2C]^2 + q_n^2 + 4C\}\, q_n\, J_0^2(q_n)}$$

$$(\partial f/\partial r)_{r=1} = -\frac{1}{r_2^2 - r_1^2} \sum_n \frac{2q_n[(q_n^2/B) - 2C]\, [r_2\, J_1(q_n r_2) - r_1\, J_1(q_n r_1)] \exp(-q_n^2 z)}{\{[(q_n^2/B) - 2C]^2 + q_n^2 + 4C\}\, J_0(q_n)}$$

$$W = \frac{1}{1 + C} - \frac{1}{r_2^2 - r_1^2} \sum_n \frac{4[(q_n^2/B) - 2C]\, [r_2\, J_1(q_n r_2) - r_1\, J_1(q_n r_1)] \exp(-q_n^2 z)}{\{[(q_n^2/B) - 2C]^2 + q_n^2 + 4C\}\, J_0(q_n)}$$

Circular distributor

$$f = \begin{cases} \infty & r = r_1 \\ 0 & 0 \leqq r < r_1 \\ 0 & r_1 < r \leqq 1 \end{cases}$$

$$2 \int_0^1 r f \,\mathrm{d}r = 1, \quad z = 0$$

$$f = \frac{C}{1 + C} + \sum_n \frac{[(q_n^2/B) - 2C]^2\, J_0(q_n r)\, J_0(q_n r_1) \exp(-q_n^2 z)}{\{[(q_n^2/B) - 2C]^2 + q_n^2 + 4C\}\, J_0^2(q_n)}$$

$$(\partial f/\partial r)_{r=1} = -\sum_n \frac{q_n^2[(q_n^2/B) - 2C]\, J_0(q_n r_1) \exp(-q_n^2 z)}{\{[(q_n^2/B) - 2C]^2 + q_n^2 + 4C\}\, J_0(q_n)}$$

$$W = \frac{1}{1 + C} - \sum_n \frac{2\,[(q_n^2/B) - 2C]\, J_0(q_n r_1) \exp(-q_n^2 z)}{\{[(q_n^2/B) - 2C]^2 + q_n^2 + 4C\}\, J_0(q_n)}$$

f_0, appearing as a scaling factor of the dimensional solution must be again replaced by the expression $Q/(\pi R^2)$.

The distribution curves computed for $B = 6{\cdot}82$ and $C = 3{\cdot}12$ (the values found[5] typical for a packing of 25 mm Raschig rings) are shown in Fig. 1*a*–*d* for all four above cases. It can be seen that the transition to the equilibrium state proceeds much faster for the circular distributor in comparison with the central point source. Similarly, the transition is faster for the annular distributor in comparison with the disc, although for the given values of radii r_1 and r_2 the maximum local density of wetting is in both cases the same. Interesting is also a comparison of the circular and the annular distributor. The radii of the latter are taken so that $(r_1 + r_2)/2 = 1/2$, which is equal to the radius of the circular distributor. From Fig. 1 it is seen that the differences of the corresponding distribution curves for both distributors are not very marked and diminish with increasing height of the packing. Thus being a relatively easy-to-realize the circular distributor may well replace the annular distributor.

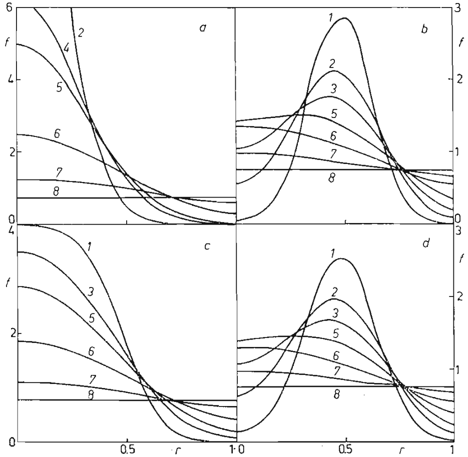

FIG. 1

Computed Distribution Curves for Several Types of Initial Wetting, ($B = 6{\cdot}82$, $C = 3{\cdot}12$)

a Central point source; *b* circular distributor $r_1 = 1/2$; *c* disc distributor $r_1 = 1/2$; *d* annular distributor $r_1 = 3/8$, $r_2 = 5/8$; *1* To = 0·01; *2* 0·02; *3* 0·03; *4* 0·04; *5* 0·05; *6* 0·10; *7* 0·2; *8* ∞.

Clearly, even faster transition to the quilibrium distribution (curve *8*) could have been achieved with a somewhat greater radius of the circular distributor, which would have avoided the situation apparent from Fig. 1*d*: For $r_1 = 1/2$ (or even smaller) the central part of the column is saturated faster to a value exceeding the equilibrium density of wetting. Consequently, a part of the liquid is later transported back across the whole column radius toward the wall.

Eqs (*3*), (*10*) and (*11*) thus enable the solutions of the distributions of the density of wetting to be found for an almost arbitrary type of initial wetting. Certain selected types of the distribution were verified experimentally: The disc distributor of $r_1 = 1/2$ and the central point source.

EXPERIMENTAL

The measurements were carried out in a glass column 291 mm in inner diameter. The packings used were glass spheres 15 and 20 mm in diameter and porcelain Raschig rings 15 and 25 mm in diameter. Water at 25°C was used in all cases. The measurements covered the range of the mean density of wetting (superficial velocity of liquid) between 0·0005 and 0·008 m/s.

As a central distributor approximating the function of a central point source we used a nozzle 20 mm in inner diameter. Further decrease of the inner diameter of the discharge opening attempting a better approximation of the hypothetic point source is not practicable because of the high

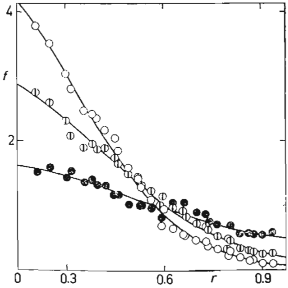

FIG. 2

A Comparison of the Computed and Experimental Profiles of the Wetting by Central "Point" Source, ($f_0 = 0{\cdot}00202$ m/s, 25 mm Raschig Rings)

○ $h = 300$ mm; ⦶ $h = 500$ mm; ● $h = 1000$ mm.

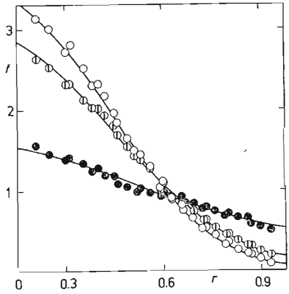

FIG. 3

A Comparison of the Computed and Experimental Profiles of the Wetting by the Disc Distributor, ($r_1 = 1/2$, $f_0 = 0{\cdot}00489$ m/s, 15 mm Raschig Rings)

○ $h = 300$ mm; ⦶ $h = 500$ mm; ● $h = 1165$ mm.

velocity of the jet impinging on the packing. In addition, the diameter of the nozzle is about the same as the characteristic dimension of the packing piece.

As a disc distributor served a brass vessel with hollow rivets mounted in its bottom evenly in a 10 × 10 mm square pitch and covering the central area equal one half of the column cross-section. There were 7 mm long nylon thread loops mounted in each rivet to improve the function of the distributor at low discharge velocities.

The method of experimental measurement of the profiles of the density of wetting is identical to that described in the preceding communication[5]. The packing rests on 13 concentric annuli and the time necessary for collecting a preselected volume of liquid is measured. Each measurement of a given depth of the packing was repeated 6 times (3 redumpings of the bed each with duplication) and the primary results were averaged (arithmetic mean of the inverse values of the times to fill the collecting vessels). Each series of experiments (9 values of liquid flow rate) was started from the maximum after flooding the column by liquid.

RESULTS AND DISCUSSION

The experimental set-up provides up to 72 points on each profile of the density of wetting. Of these we selected 30 points having the width of the collecting annulus at least comparable[6] (but in most cases greater) than the characteristic dimension of the packing.

Since the distribution of the density of wetting for the selected types of initial distribution is strongly non-uniform it must be expected that the agreement between the experimental and computed profiles will be markedly affected by the choice of the parameter D necessary for evaluating the dimensionless height of the packing To. We have therefore compared the experimental profiles with the computed curves for such To which satisfies the condition of the minimum sum of square deviations from the experimental profile. For B and C we took $B = 6{\cdot}82$ (for all packings) and $C = 8{\cdot}44$ and $6{\cdot}32$ for spheres 15 and 20 mm in diameter, and $C = 3{\cdot}66$ and $3{\cdot}12$ for Raschig rings 15 and 25 mm in diameter. These values were reported in the preceding paper[5]. The agreement found was in all cases very good without systematic deviations. Typical computed curves and experimental points are shown in Fig. 2 for 25 mm Raschig rings wetted by central "point" source. A similar situation is shown in Fig. 3 for 15 mm Raschig rings wetted by the disc $(r_1 = 1/2)$ distributor. The good agreement between the experimental and predicted values of the density of wetting in case of the central source is somewhat surprising because the coefficients D computed from the optimized values To tend to increase with f_0, particularly on low layers of packing. This is caused by the already earlier observed flooding of the packing on the top where large quantities of liquid are discharged on a small area. The dependence of D on f_0 would of course deny the linearity of the problem and hence the validity of Eq. (*1*). The non-linearity of the problem, however, is refuted by the fact that the standard deviations found by comparing the experimental and predicted profiles display only a very small increase with f_0 in case of the spheres, while for 25 mm Raschig rings they tend to decrease. The increase of D with f_0 can be explained by the observation suggesting that the flooding of the packing occurs immediately on the top of the packing layer which is not able to absorb all supplied liquid. The excess liquid spreads over the top on a larger area until its velocity decreases below the value that can be absorbed by the packing. The central "point" source thus transforms into a disc distributor the radius of which depends on f_0. For discs of small radius and larger values of To the solution differs little from that for the central point source which explains the good agreement of the experimental and theoretical profile under seemingly non-linear (judging from D) conditions.

LIST OF SYMBOLS

A_n	coefficient of expansion
B	dimensionless parameter of the boundary condition (*2*)
C	dimensionless parameter of the boundary condition (*2*)
D	coefficient of radial spreading of liquid, (L)
d_p	characteristic dimension of the packing, (L)
$f = f'/f_0, f'$	dimensionless and dimensional, (LT^{-1}), density of wetting
$f_0 = Q/(\pi R^2)$	mean density of wetting, (LT^{-1})
h	height of bed, (L)
J_0, J_1	Bessel function of the first kind, zero and first order
n	summation index
Q	amount of liquid brought into column per unit time (L^3T^{-1})
q_n, q_m	roots of Eq. (*4*)
R	column diameter, (L)
$r = r'/R, r'$	dimensionless and dimensional, (L) radial coordinate
$r_1 = r'_1/R, r_2 = r'_2/R, r'_1, r'_2$	dimensionless and dimensional, (L), radii
$\text{To} = Dh/R^2$	dimensionless height of bed
$W = W'/Q, W'$	dimensionless and dimensional, (L^3T^{-1}), wall flow
$z = Dz'/R^2, z'$	dimensionless and dimensional, (L), coordinate of height
$\varphi(r)$	function determining the initial distribution of liquid

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Translated by V. Staněk.